Draw and interpret curved graphs

Quadratic Equations

An equation which has the form:

$y = ax^2 + bx + c$

where **a** is not zero and the highest power of *x* is two (x^2) is called a **quadratic** equation. The constants **b** and **c** can take any numerical value.
When the graph of a quadratic equation is plotted, it always gives a smooth curve called a parabola. (It is non-linear.)
Because the required graph is a curve we need to find more pairs of coordinates of points in order to enable us to construct the curve with reasonable accuracy.

Example

a. The graph of $y = x^2$.

If we take *x* values from -4 to 4 and calculate corresponding values of *y* we get the following table of results:

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
y	16	9	4	1	0	1	4	9	16

Plotting the points and joining them with a smooth curve gives us the graph of $y = x^2$

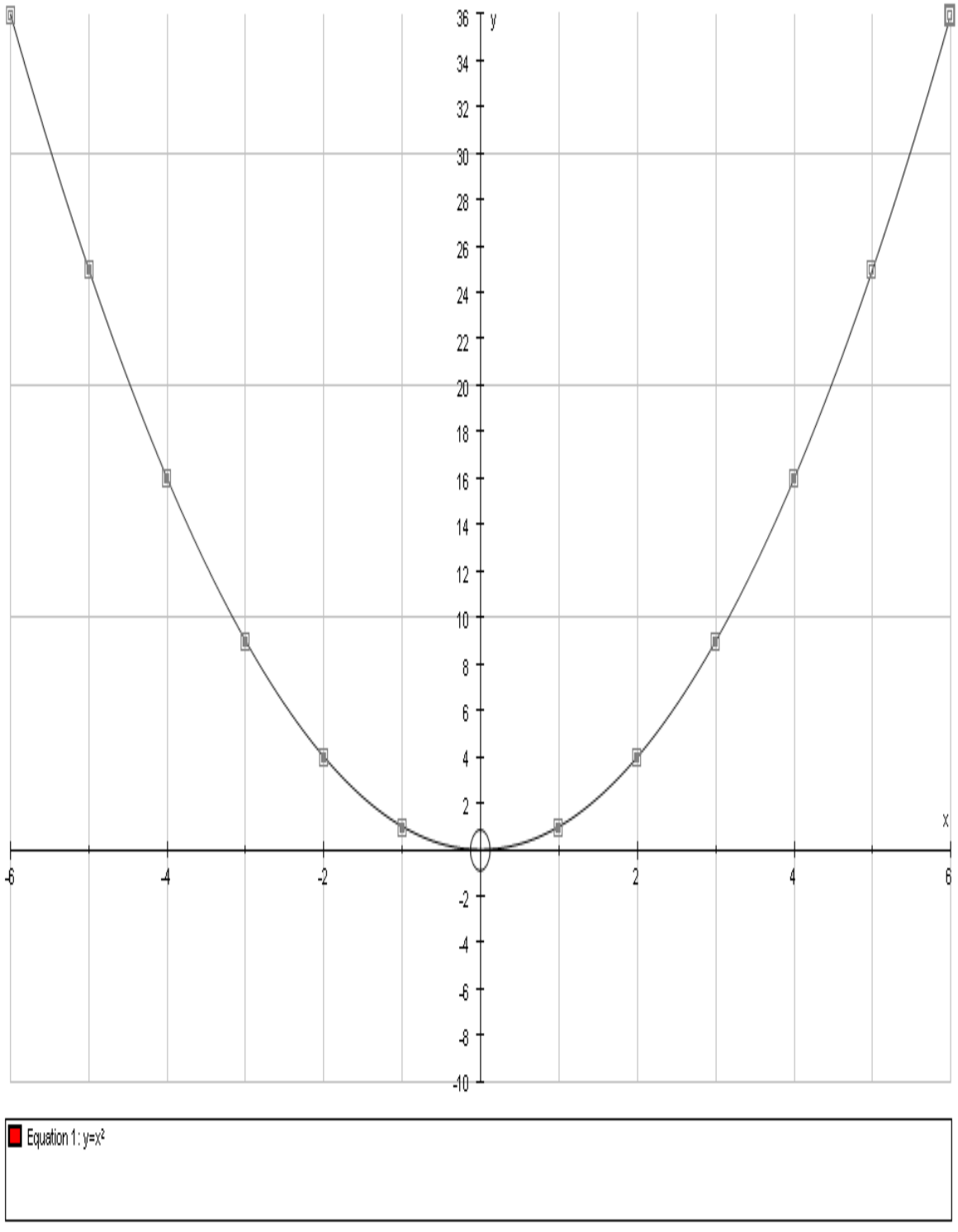
Notice that the vertical scale and horizontal scale differ. This will usually be the case. Try to ensure that the graph is as big as you can feasibly make it on your graph paper. Big is beautiful.

Worked Examples

b. Using the values of *x* from -4 to 4, graph the equation **$y = x^2 - 5$**.
Complete the table of values:

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9			0	1	4		
- 5	-5	-5	-5	-5	-5	-5	-5	-5	-5
y	11	4			-5	-4	-1		

Try to make your curve smooth.


y
x
36
34
32
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28
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18
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10
8
6
4
2
-2
-4
-6
-8
-10
-6
-4
-2
2
4
6
Equation 1: y=x²

c. Using the values of x from -4 to 4, graph the equation **$y = 2x^2$**.
Complete the table of values:

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16					1			
$2x^2$	32	18			0	2	8		
y	32	18			0	2	8		

d. Drawing quadratic graphs of the form $y = ax^2 + bx + c$

A curve has equation **$y = x^2 - 2x - 8$**

Complete the table of values:

x	-3	-2	-1	0	1	2	3	4	5
x^2	9	4				4	9	16	25
- 2x	6	4			-2	-4	-6		
-8	-8	-8	-8	-8	-8	-8	-8	-8	-8
y	7	0				-8			

Then plot the points and join them up.

The solutions of **$0 = x^2 - 2x - 8$** can be read off the graph. They are the two points where $y = 0$. i.e. $x =$ and $x =$

Find the minimum point x = , y =

Find the value of y when x = 4.5 y =

Find the value of x when y = - 2.5 x= , x =

Exercises 1

1. Using the values of x from -4 to 4, graph the equation $y = 2x^2 - 5$

First we make a table of values

x	–4	–3	–2	–1	0	1	2	3	4
x^2		9					4		
$2x^2$		18					8		
–5	-5	-5					-5		
y		13					3		

Then plot the points and join them up.

The solutions of $0 = 2x^2 - 5$ can be read off the graph. They are the two points where $y = 0$. i.e. $x =$ and $x =$

2. A curve has equation **$y = x^2 + 5x + 2$**

Complete the table of values:

x	–6	–5	–4	-3	-2	-1	0	1
x^2	36	25		9		1		1
+5x	-30				-10	-5		
+2	2	2	2					
y	8							

Then plot the points and join them up.

The solutions of **$0 = x^2 + 5x + 2$** can be read off the graph. They are the two points where $y = 0$. i.e. $x =$ and $x =$

Find the minimum point x = , y =

Find the value of y when x = - 1.5 y =

3. A curve has equation $\mathbf{y = - x^2 + 5x - 4}$
Complete the table of values:

x	0	1	2	3	4	5
$-x^2$	0					-25
+5x	0					25
-4	-4					-4
y	-4					-4

Then plot the points and join them up.

The solutions of $\mathbf{0 = - x^2 + 5x - 4}$ can be read off the graph. They are the two points where $y = 0$. i.e. $x =$ and $x =$ –

Estimate the maximum point x = , y =

Estimate the value of x when y = 1 x =

Practical Applications

Having used data from experiments to construct graphs, we then need to be able to use the graphs to estimate other solutions not given directly by those experiments.

Exercise

Mike throws a tennis ball up in the air.
Its height, *h* metres, at time *t* seconds is given by
$h = 20\,t - 5\,t^2$

t	0	1	2	3	4
20t	0	20			80
$\mathbf{-5\,t^2}$	0	-5			-80
h					

a) Complete this table of values of *h*
b) Chose suitable scales and draw the graph of *h* against *t*
c) Use your graph to estimate the times at which the ball is 10 m above the ground. Why do you get two answers?
d) Why is not sensible to take values of t greater than 4?