Compound Interest

People can save money by putting it in a deposit account or a savings account in a bank or building society. They receive a payment called interest. Interest is paid at a certain percent per year.

***Example** 1:* 5% interest on £1000 savings.

Method: $\frac{5}{100^{1}} \times \frac{1000^{10}}{1} = \frac{5}{1} \times \frac{10}{1} = \frac{50}{1}$ = £50 interest at the end of the year

If you withdraw the interest and leave the savings (called the principal) in the account you will earn another £50 in the second year – total £100.

After three years at £50 each year the interest totals £150.

This way of payment, when the interest paid is not invested, called **simple interest.**

With **compound interest**, the amount of interest paid is reinvested and earns interest itself.

In compound interest, the principal changes every year, as the previous year interest is added into it.

Example 2:

£1000 is invested at 5% per annum compound interest. What is the amount after 3 years and what is total interest received in 3 years?

Year 1. Interest (I) = 1000 * 5 / 100 = £50
Total amount (A) at the account at the end of Year 1 is
A = 1000 + 50 = £1050

Year 2. I = 1050 * 5 / 100 = £52.50
Total amount (A) at the account at the end of Year 2 is
A = 1050 + 52.50 = £1102.50

Year 3. I = 1102.50 * 5 / 100 = £55.125
Total amount (A) at the account at the end of Year 3 is
A = 1102.50 + 55.125 = £1157.625 = £1157.63 (to the nearest penny).

The amount of compound interest got is:
I = 1157.63 – 1000 = £157.63

The formula for compound interest is:

$$A = P\left(1 + \frac{R}{100}\right)^T$$

where: A = total amount
P = original investment (principal)
R = rate % per annum
T = time in years

Using this formula for Example 2:

P = original investment (principal) = 1000
R = rate % per annum = 5
T = time in years = 3

$A = 1000\left(1 + \frac{5}{100}\right)^3 = 1000 * 1.05^3 = £1157.63$ (as before).

Comparing the results of examples 1 and 2 we can see that at the same given conditions – principal, rate and number of years - amount of compound interest is bigger.

This topic regularly appears on the exam paper (usually calculator). The tasks could be straightforward as Example 2, asking to workout the amount of money after given number of years or reverse as the next example from *EDEXCEL* calculator November 2003 paper.

***Example 3*:**

Henry invests £4500 at a compound interest rate of 5% per annum.
At the end of **n** complete years the investment has grown to £5469.78.
Find the value of **n**.

First way of solving this task can be just finding the interest received and total amount of money after each year:

Year 1: Interest received after the 1st year
I = 4500 * 5 / 100 = 225
Total amount A = 4500 + 225 = 4725

Year 2: I = 4725 * 5 / 100 = 236.25
A = 4725 + 236.25 = 4961.25

Year 3: I = 4961.25 * 5 / 100 = 248.0625
A = 4961.25 + 248.0625 = 5209.3125

Year 4: I = 5209.3125 * 5 / 100 = 260.46562
A = 5209.3125 + 260.46562 = 5469.7781 = £5469.78

So it took **n** = 4 years for the investment to grow up to the asking sum.

Another solution for this task is to take a good guess of **n** using the formula.

Let **n** = 3:
P = 4500, R = 5, T = n = 3

$$A = 4500 \left(1 + \frac{5}{100}\right)^3 = 4500 * (1+0.05)^3 = 4500 * 1.157625 =$$

= 5209.3125 (as before) which is less than we need.

So let us try **n** = 4

$$A = 4500 \left(1 + \frac{5}{100}\right)^4 = 4500 * 1.2155 = 5469.7781 = £5469.78$$

which is the right answer.